Logics of propositional control for social choice

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Logics for social choice

- social software [Parikh 2001]: give to **societies of agents** what the theory of computation is to computers.
- why logic?
  - close to reasoning/computations;
  - axiomatic method.
Strategic games

- simple abstraction of agent interaction;
- a strategic game specifies a set of consequences and for each player:
  - a set of possible actions;
  - a preference ordering over the set of consequences.
and an outcome function mapping every action profile to a consequence.
- solution concept, equilibrium: function mapping a strategic game to a set of action profiles.
Objectives

A logic for reasoning about strategic games:
- \textit{characterising} solution concepts;
- \textit{finding} equilibria (model checking);
- \textit{checking} game solvability (theorem proving).

A logic for reasoning about social choice functions (e.g., voting procedures).

... by means of \textit{propositional control}.
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**Outline**

1. Logic

2. Immediate examples of application

3. Application to social choice functions
Background: propositional control

(ATL model checking [Alur et al. 2000]; Boolean games: [Harrenstein et al. 2001]; Coalition Logic of Propositional Control [van der Hoek, Wooldridge 2005], [Gerbrandy 2006])

A frame of propositional control consists of:

- a set of controlled atoms $A_t$;
- a set of players $N$;
- a set of controlled atoms $A_{t_i}$ for every player $i$.

Intuitively:

- action $a_i$ of $i \in N \iff$ valuation $\theta_i$ of $A_{t_i}$;
- action profile $a_N \iff$ valuation $\theta_N$ of $A_t$. 
GPCC: Models

Definition

A game of propositional control with consequences is a tuple \( \langle N, At, (At_i), K, o, (\leq_i) \rangle \), such that:

- \( \langle N, At, (At_i) \rangle \) is a frame of propositional control;
- \( K \) is a nonempty finite set of atoms such that \( At \cap K = \emptyset \);
- \( o \) maps a \( \theta_N \) valuation to an element of \( K \);
- \( \leq_i \) is a preference relation over \( K \) for every agent \( i \).
GPCC: Language

Parameters:

- $N$: a set of players;
- $At$: a set of controlled atoms;
- $K$: a set of atoms of Konsequences.

The language $\mathcal{L}(N, At, K)$ is inductively defined by the following grammar:

$$\varphi ::= T \mid a \mid \neg \varphi \mid \varphi \lor \varphi \mid \Diamond C \varphi \mid \langle \leq i \rangle \varphi$$

where $a$ is an atom of $At \cup K$, $C \subseteq N$ is a coalition and $i$ is a member of $N$. 
GPCC: Truth values

\[ G = \langle N, At, (At_i), K, o, (\preceq_i) \rangle, \theta_N \text{ is a valuation of } At. \]

\begin{align*}
G, \theta_N \models p & \iff \theta_N(p) = \text{tt} & p \in At \\
G, \theta_N \models x & \iff o(\theta_N) = x & x \in K \\
G, \theta_N \models \Diamond_C \varphi & \iff \text{there is a valuation } \theta_N' \text{ such that } \\
& G, \theta_N' \models \varphi \text{ and } \forall j \in N \setminus C : \theta_{N}'_j = \theta_j \\
G, \theta_N \models \langle \preceq_i \rangle \varphi & \iff \text{there is a valuation } \theta_N' \text{ such that } \\
& G, \theta_N' \models \varphi \text{ and } o(\theta_N) \preceq_i o(\theta_N')
\end{align*}

Readings:
- \( \Diamond_C \varphi \): the players in \( C \) can deviate from their current strategy such that \( \varphi \).
- \( \langle \preceq_i \rangle \varphi \): agent \( i \) would prefer \( \varphi \).


Defined vocabulary

- Reifying the valuations (for every valuation $\theta_N \in \Theta$):
  \[
  \pi(\theta_N) \triangleq \bigwedge_{\theta_N(p)=tt} p \land \bigwedge_{\theta_N(q)=ff} \neg q.
  \]

- An operator of binary preferences (for every player $i \in N$):
  \[
  \psi \leq^i_{VV} \varphi \triangleq \Box_N \bigvee_{\theta_N \in \Theta} (\pi(\theta_N) \land (\varphi \rightarrow \Box_N(\psi \rightarrow \langle \leq_i \rangle \pi(\theta_N)))).
  \]
A first example

Consider a model $G$ with $N = \{1, 2\}$, $At_1 = \{p_1, q_1\}$, $At_2 = \{p_2, q_2\}$, $K = \{x, y, z\}$. Suppose $z \prec_2 x \prec_2 y$.

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<thead>
<tr>
<th>$\neg p_2 \land \neg q_2$</th>
<th>$p_2 \land \neg q_2$</th>
<th>$\neg p_2 \land q_2$</th>
<th>$p_2 \land q_2$</th>
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<tbody>
<tr>
<td>$\neg p_1 \land \neg q_1$</td>
<td>$x \bullet$</td>
<td>$x \bullet$</td>
<td>$z \bullet \theta'_N$</td>
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<td>$p_1 \land \neg q_1$</td>
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<td>$\neg p_1 \land q_1$</td>
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<td>$y \bullet \theta_N$</td>
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<td>$z \bullet$</td>
<td>$z \bullet$</td>
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We have:

- $G, \theta_N \models \Diamond_1 \pi(\theta'_N)$
- $G, \theta_N \models \Diamond_1 x$
- $G, \theta'_N \models \langle \leq_2 \rangle \pi(\theta_N)$
- $G, \theta_N \models z \leq_2^\forall \forall x$
GPCC: Axiomatics

**CL-PC** [Gerbrandy 2006]

(Prop) \( \varphi \), where \( \varphi \) is a propositional tautology

(K(i)) \( \Box_i(\varphi \rightarrow \psi) \rightarrow (\Box_i\varphi \rightarrow \Box_i\psi) \)

(T(i)) \( \Box_i\varphi \rightarrow \varphi \)

(B(i)) \( \varphi \rightarrow \Box_i\varphi \)

(comp) \( \Box C_1 \Box C_2 \varphi \leftrightarrow \Box (C_1 \cup C_2 \varphi) \)

(empty) \( \Box_0 \varphi \leftrightarrow \varphi \)

(exclu) \( (\Diamond_i p \land \Diamond_i \neg p) \rightarrow (\Box_j p \lor \Box_j \neg p) \), where \( j \neq i \)

(actual) \( \bigvee_{i \in \mathbb{N}} \Diamond_i p \land \Diamond_i \neg p \)

(full) \( (\bigwedge_{p \in X} \Diamond_i p \land \Diamond_i \neg p) \rightarrow \Diamond_i \varphi_v \), where \( \varphi_v \) is the conjunction of literals true in one valuation \( v \) of \( X \subseteq \text{At} \)

**Outcomes and preferences**

(func1) \( \bigvee_{x \in K} (x \land \bigwedge_{y \in K \setminus \{x\}} \neg y) \)

(func2) \( (\pi(\theta_N) \land x) \rightarrow \Box_N(\pi(\theta_N) \rightarrow x) \)

(incl) \( \Box N \varphi \rightarrow [\leq_i] \varphi \)

(K(\leq_i)) \( [\leq_i](\varphi \rightarrow \psi) \rightarrow ([\leq_i] \varphi \rightarrow [\leq_i] \psi) \)

(4(\leq_i)) \( [\leq_i](\leq_i) \varphi \rightarrow (\leq_i) \varphi \)

(connect) \( (\varphi \land \Diamond_N \psi) \rightarrow (\leq_i) \psi \lor \Diamond_N (\psi \land (\leq_i) \varphi) \)

(unifPref) \( (x \land (\leq_i) y) \rightarrow x \leq_{i,v} y \)

**Rules**

(MP) from \( \vdash \varphi \rightarrow \psi \) and \( \vdash \varphi \) infer \( \vdash \psi \)

(Nec(\Box_i)) from \( \vdash \varphi \) infer \( \vdash \Box_i \varphi \)
Outline

1. Logic
2. Immediate examples of application
3. Application to social choice functions
Characterising equilibria: e.g. Nash equilibrium

Equilibrium: property of action profile $\iff$ property of valuation

Weak best response by the agent $i$:

$$WBR_i \triangleq \bigvee_{x \in K} (x \land \Box_i(\preceq_i x))$$

Nash equilibrium:

$$NE \triangleq \bigwedge_{i \in N} WBR_i$$

And many more solution concepts.
Finding equilibria (by model checking)

Model checking [Clarke, Grumberg, Peled 1999]:
- Input: a model $M$ and a formula $\varphi$;
- Output: $\{s \in S : M, s \models \varphi\}$.

Here, e.g. finding Nash equilibria:
- Input: a GPCC $G$ and the formula $NE$;
- Output: $\{\theta_N \in \Theta : G, \theta_N \models NE\}$. 
Game solvability (by theorem proving)

The “wedding scenario” [Gibbard, 1974]:

- Three players: Angelina ($a$), Edwin ($e$) and the male Judge ($j$);
- Angelina can choose to get married ($p'_a$);
- Angelina can choose to marry Edwin ($p_a$) or the Judge ($\neg p_a$);
- Edwin (resp. the Judge) can choose to marry Angelina ($p_e$ (resp. $p_j$)) or to stay single;
- The consequences are that Angelina marries Edwin ($m_e$) (resp. the Judge ($m_j$)) when both of them choose so; otherwise she stays single ($s$).
Game solvability (by theorem proving)

- encoding the controls:
  \[ \rho \triangleq (\diamond a p_a \land \diamond a \neg p_a \land \diamond a p'_a \land \diamond a \neg p'_a) \land (\diamond e p_e \land \diamond e \neg p_e) \land (\diamond j p_j \land \diamond j \neg p_j). \]

- encoding the outcome function:
  \[ \omega_{m_e} \triangleq m_e \leftrightarrow (p'_a \land p_a \land p_e). \quad \text{(marrying Edwin)} \]
  and
  \[ \omega_{m_j} \triangleq m_j \leftrightarrow (p'_a \land \neg p_a \land p_j). \quad \text{(marrying the Judge)} \]
We can verify that this scenario has a Nash solvable representation, stated by the validity of

$$\rho \land \omega_{m_e} \land \omega_{m_j} \rightarrow \diamond_NNE$$

in the logic $\Lambda(\{a, e, j\}, \{p_a, p'_a, p_e, p_j\}, \{s, m_e, m_j\})$. 

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‘natural’ logic for reasoning about problems of strategic games;
expressive language: Pareto optimality, dominance, strong Nash equilibrium, core membership...;
decidable logic;
application to:
  finding strategic equilibria;
  game solvability;
  implementation theory (→ next slides)
  [Troquard, van der Hoek, Wooldridge (TARK’09)].
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Social choice functions as strategic game forms:

- A social choice function associates a consequence to every preference profile. (e.g., voting procedure.)
- Mathematically equivalent to a strategic game form where the set of actions for every agent is the set of preferences over $K$.

We design a logic similar to previously, except:

- Preferences are linear order (for simplicity);
- Valuations of controlled atoms encode a preference profile.
Definitions and notations

- $N$ is the set of players; $K$ is the set of consequences;
- $L(K)$ is the set of linear orderings over $K$;
- $At[i, K] = \{ p^i_{x>y} \mid x, y \in K \}$ is the set of atoms controlled by $i$;
- $At[C, K] = \bigcup_{i \in C} At[i, K]$ is the set of atoms controlled by $C$;
- $\text{strategies}[i, K]$ is the set of valuations of $At[i, K]$ such that it “encodes a linear order”: strategies of $i$;
- for every coalition $C \subseteq N$, we note $\text{strategies}[C, K]$ the set of tuples $v_C = (v_i)_{i \in C}$ where $v_i \in \text{strategies}[i, K]$. 


Intuitively...

- an action of $i \in N \iff$ a linear preference over $K \iff$ a valuation in $\text{strategies}[i, K]$
- a strategy profile $\iff$ a preference profile $\iff$ valuation in $\text{strategies}[N, K]$
The language $\mathcal{L}^{scf}[N, K]$ is inductively defined by the following grammar:

$$\varphi ::= \top | p | x | \neg \varphi | \varphi \lor \varphi | \Diamond C \varphi | \langle <i \rangle \varphi.$$ 

where $p$ is atom of $\text{At}[N, K]$, $x$ is an atom of $K$, $i \in N$, and $C$ is a coalition.
Models

Definition (model of social choice functions)

A *model of social choice functions over* $N$ and $K$ *is a tuple* $M = \langle N, K, out, (<_i) \rangle$, *such that:*

- $N = \{1, \ldots, n\}$ is a finite nonempty set of players;
- $K$ is a finite nonempty set of consequences;
- $out : strategies[N, K] \rightarrow K$ maps every strategy profile to a consequence;
- For every $i \in N$, $<_i \in L(K)$ is the true preferences of $i$. 
Definition (truth values of $L^{scf}[N, K]$)

Given a model $M = \langle N, K, out, (\prec_i) \rangle$, we are going to interpret formulas of $L^{scf}[N, K]$ in a preference profile $v \in \text{strategies}[N, K]$ of the model. The truth definition is inductively given by:

- $M, v \models p$ iff $p \in v_i$ for some $i \in N$
- $M, v \models x$ iff $\text{out}(v) = x$
- $M, v \models \neg \varphi$ iff $M, v \not\models \varphi$
- $M, v \models \varphi \lor \psi$ iff $M, v \models \varphi$ or $M, v \models \psi$
- $M, v \models \Diamond_C \varphi$ iff there is a $u \in \text{strategies}[N, K]$ such that $v_i = u_i$ for every $i \notin C$ and $M, u \models \varphi$
- $M, v \models (\prec_i) \varphi$ iff there is a $u \in \text{strategies}[N, K]$ such that $\text{out}(v) \prec_i \text{out}(u)$ and $M, u \models \varphi$
Ballots: propositional encoding

**Definition (ballot)**

For every player \( i \in N \), we can see every \( <_i \in L(K) \) as a permutation \([x_1, x_2 \ldots]\) of the elements of \( K \), where the more to the left the consequence is, the more it is preferred by the player \( i \). We can reify in the language the reported preferences, that is, the ballot casted by the player \( i \):

\[
\text{ballot}_i(<) \triangleq p^i_{x_1>x_2} \land p^i_{x_2>x_3} \land \ldots p^i_{x_{|K|-1}>x_{|K|}}.
\]

Then, the formula

\[
\text{ballot}(<) \triangleq \bigwedge_{i \in N} \text{ballot}_i(<)
\]

is a reification of the reported preference profile \(< = (<_1, \ldots, <_n)>\), consisting of one ballot for every player \( i \in N \).
# Characterising social choice functions

## Definition (SCF characterisation)

We say that the formula $\rho^F \in \mathcal{L}^{scf}[N, K]$ characterises the social choice function $F$ if for all $\prec \in L(K)^N$ and $x \in K$ we have:

$$F(\prec) = x \ \text{iff} \ \models_{\Lambda^{scf}[N,K]} \rho^F \rightarrow \Diamond_N(\text{ballot}(\prec) \land x).$$

## Remark (expressively complete)

For every social choice function $F$ there is always such a characteristic formula $\rho^F$. 
True preferences

The agent $i$ prefers the proposition $\psi$ over $\varphi$:

$$\psi \preceq_i \varphi \triangleq \square_N \bigvee_{< \in L(K)^N} (\text{ballot}(<) \land (\varphi \rightarrow \square_N(\psi \rightarrow \langle i \rangle \text{ballot}(<)))).$$ 

Definition (true preferences)

from a preference profile $< \in L(K)^N$, we reify the preference $[x_1, x_2 \ldots]$ of the player $i$ as follows:

$$\text{true}_i(<) \triangleq (x_{|K|} \preceq_i x_{|K|-1}) \land \ldots \land (x_3 \preceq_i x_2) \land (x_2 \preceq_i x_1).$$

Then, the formula

$$\text{true}(<) \triangleq \bigwedge_{i \in N} \text{true}_i(<)$$

is a reification of the true preference profile $<= (<_1, \ldots, <_n)$. 

Λ(\(N, At, K\)): Axiomatics

<table>
<thead>
<tr>
<th>Constraints of control</th>
<th>(p_x^{i,x})</th>
<th>(p_x^{i,y} \leftrightarrow \neg p_y^{i,x}), where (x \neq y)</th>
<th>(p_x^{i,y} \land p_y^{i,z} \rightarrow p_x^{i,z})</th>
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<tr>
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<td>(p_x^{i,y} \land p_y^{i,z} \rightarrow p_x^{i,z})</td>
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<td>(antisym-total)</td>
<td>(p_x^{i,y} \leftrightarrow \neg p_y^{i,x}), where (x \neq y)</td>
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<td>(trans)</td>
<td>(p_x^{i,y} \land p_y^{i,z} \rightarrow p_x^{i,z})</td>
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<thead>
<tr>
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<th>(\varphi), where (\varphi) is a propositional tautology</th>
<th>(\Box_i(\varphi \rightarrow \psi) \rightarrow (\Box_j \varphi \rightarrow \Box_j \psi))</th>
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<td>(K(i))</td>
<td>(\Box_i \varphi \rightarrow \varphi)</td>
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<td>(T(i))</td>
<td>(\square \varphi \rightarrow \varphi)</td>
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<td>(B(i))</td>
<td>(\varphi \rightarrow \square_i \varphi)</td>
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<tr>
<td>(comp∪)</td>
<td>(\Box C_1 \Box C_2 \varphi \leftrightarrow \Box (C_1 \cup C_2) \varphi)</td>
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<td>(empty)</td>
<td>(\square \varphi \leftrightarrow \varphi)</td>
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<tr>
<td>(exclu)</td>
<td>(\Diamond_i p \land \Diamond_j \neg p \rightarrow (\Diamond_j p \lor \Diamond_j \neg p)), where (j \neq i)</td>
<td></td>
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<tr>
<td>(ballot)</td>
<td>(\Diamond_i \text{ballot}(\langle\rangle))</td>
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<tr>
<td>(comp-At)</td>
<td>(\Diamond C_1 \delta_1 \land \Diamond C_2 \delta_2 \rightarrow \Diamond (C_1 \cup C_2) (\delta_1 \land \delta_2))</td>
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<thead>
<tr>
<th>Consequences and preferences</th>
<th>(\bigvee_{x \in K}(x \land \bigwedge_{y \in K \setminus {x}} \neg y))</th>
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<tr>
<td>(func1)</td>
<td>(\bigvee_{x \in K}(x \land \bigwedge_{y \in K \setminus {x}} \neg y))</td>
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<td>(func2)</td>
<td>((\text{ballot}(\langle\rangle) \land \varphi) \rightarrow \Box N (\text{ballot}(\langle\rangle) \rightarrow \varphi))</td>
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<tr>
<td>(incl)</td>
<td>(\square N \varphi \rightarrow \langle i \rangle \varphi)</td>
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<tr>
<td>(K(&lt;i)))</td>
<td>([&lt;i](\varphi \rightarrow \psi) \rightarrow ([&lt;i] \varphi \rightarrow [&lt;i] \psi))</td>
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<tr>
<td>(4(&lt;i)))</td>
<td>(\langle i \rangle (&lt;i \rangle \varphi \rightarrow (&lt;i \rangle \varphi))</td>
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<tr>
<td>(antisym')</td>
<td>((\text{ballot}(\langle\rangle) \land (&lt;i \rangle \text{ballot}(\langle\rangle)) \rightarrow \Box_N (\text{ballot}(\langle\rangle) \rightarrow [&lt;i \rangle \neg \text{ballot}(\langle\rangle)))</td>
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<tr>
<td>(total')</td>
<td>((\text{ballot}(\langle\rangle) \land (&lt;i \rangle \text{ballot}(\langle\rangle)) \lor \Box_N (\text{ballot}(\langle\rangle) \rightarrow (i \rangle \text{ballot}(\langle\rangle)))</td>
</tr>
<tr>
<td>(unifPref)</td>
<td>((x \land (&lt;i \rangle y) \rightarrow (x \downarrow_i y))</td>
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<thead>
<tr>
<th>Rules</th>
<th>(\vdash \varphi \rightarrow \psi) and (\vdash \varphi) infer (\vdash \psi)</th>
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<tr>
<td>(MP)</td>
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<tr>
<td>(Nec((\Box_i)))</td>
<td>(\vdash \varphi) infer (\vdash \Box_i \varphi)</td>
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**Some properties of social choice functions**

<table>
<thead>
<tr>
<th>Concept</th>
<th>Definition</th>
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<tbody>
<tr>
<td>Citizen sovereignty</td>
<td>$\text{CITSOV} \triangleq \bigwedge_{x \in K} \Diamond_N x$</td>
</tr>
<tr>
<td>Non dictatorship</td>
<td>$\text{NODICT} \triangleq \bigwedge_{i \in N} \Diamond_N \left( \bigvee_{x \in K} \left( x \land \bigvee_{y \in K \backslash {x}} p_{y &gt; x}^i \right) \right)$</td>
</tr>
<tr>
<td>$i$’s best response</td>
<td>$\text{BR}<em>i \triangleq \bigvee</em>{x \in K} \left( x \land \Box_i \langle &lt;_i \rangle x \right)$</td>
</tr>
<tr>
<td>Dominance equilibrium</td>
<td>$\text{DOM} \triangleq \bigwedge_{i \in N} \Box_{N \backslash {i}} \text{BR}_i$</td>
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<tr>
<td>Strategy-proofness</td>
<td>$\text{STRPROOF} \triangleq \bigwedge_{&lt; \in L(K)^N} \left[ \text{true}(&lt;) \rightarrow (\text{ballot}(&lt;) \rightarrow \text{DOM}) \right]$</td>
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</tbody>
</table>
The logics we have presented are:
- axiomatised;
- decidable;
- expressive.

Future work:
- compact languages;
- integrate epistemic/doxastic notions.